

# Chaplygin gas Braneworld Inflation according to WMAP7

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## Abstract

We consider a Chaplygin gas model with an exponential potential in framework of Braneworld inflation. We apply the slow-roll approximation in the high energy limit to derive various inflationary spectrum perturbation parameters. We show that the inflation observables depend only on the e-folding number  $N$  and the final value of the slow roll parameter  $\varepsilon_{end}$ . Whereas for small running of the scalar spectral index  $\frac{dn_s}{d \ln k}$ , the inflation observables are in good agreement with recent WMAP7 data.

Keywords: Chaplygin gas, *RS Braneworld*, *Perturbation Spectrum*, *WMAP7*.

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## Contents

<b>1</b>	<b>Introduction</b>	<b>2</b>
<b>2</b>	<b>Chaplygin gas Braneworld inflation</b>	<b>3</b>
2.1	Slow-Roll approximation . . . . .	3
2.2	Chaplygin gas perturbation spectrum . . . . .	4
<b>3</b>	<b>Exponential potential in high energy limit</b>	<b>5</b>
<b>4</b>	<b>Conclusion</b>	<b>8</b>

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# 1 Introduction

In the last few years, Braneworld inflation[1] became a central paradigm of cosmology. In this context, *Randall-Sundrum* type 2 model[2] has attracted a lot of interests. To describe early inflation and recent acceleration of the universe, various Braneworld cosmological models have been proposed[3] and deeply studied[4]. Recently, a new Chaplygin gas model[5] was introduced to describe dark matter and dark energy recently discovered[6]. Chaplygin gas model is a particular kind of matter, characterized by an exotic equation of state and was applied to study various cosmological models, such as chaotic canonical scalar field inflation[7] and exponential tachyonic inflation on the brane[8]. More recently, a generalized version of the original Chaplygin gas model has been proposed and studied in the context of late time acceleration of the Universe[9] and in the context of Braneworld model[10]. Note that Chaplygin gas has also recently been used to describe early inflationary universe[11]. In the Chaplygin gas inspired inflation model, the scalar field is usually the standard inflaton field, where the energy density can be extrapolated to obtain a successful inflationary period with a Chaplygin gas model[12]. The motivation for introducing Chaplygin gas Braneworld models is the increasing interest in studying dark matter and dark energy in higher-dimensional cosmological models. We signal in this context, that Chaplygin gas model is considered now as a viable alternative model that can provide an accelerated expansion of the early universe. The Chaplygin gas emerges as an effective fluid of a generalized d-brane in space time, where the action can be written as a generalized Born-Infeld action[13].

In this work, our aim is to quantify the modifications of the Chaplygin gas inspired inflation in the Braneworld scenario. Recall that the generalized Chaplygin gas is defined by an exotic equation of state of the form[13]

$$p_{ch} = -\frac{A}{\rho_{ch}^\alpha}, \quad (1)$$

where  $\rho_{ch}$  and  $p_{ch}$  are the energy density and pressure of the generalized Chaplygin gas, respectively.  $\alpha$  is a constant satisfying  $0 < \alpha \leq 1$ , and  $A$  is a positive constant. From the matter conservation equation, one can obtain the following generalized Chaplygin gas energy density expression

$$\rho_{ch} = \left[ A + \left( \rho_{ch_0}^{\alpha+1} - A \right) \left( \frac{a_0}{a} \right)^{3(\alpha+1)} \right]^{\frac{1}{\alpha+1}}, \quad (2)$$

where  $a_0$  and  $\rho_{ch_0}$  are the present day values of the scale factor and the generalized Chaplygin gas energy density, respectively. Note that the original Chaplygin gas model corresponds to  $\alpha = 1$ . In this case, it was shown that the generalized Chaplygin gas model can describe late time acceleration of the Universe either for small values of the parameter  $\alpha$  or for very large ones[14]. In order to describe early inflationary universe, we can use the following extrapolation[12]

$$\rho_{ch} = \left[ A + \rho_m^{(\alpha+1)} \right]^{\frac{1}{\alpha+1}} \longrightarrow \rho_{ch} = \left[ A + \rho_\phi^{(\alpha+1)} \right]^{\frac{1}{\alpha+1}}. \quad (3)$$

In this work, we first start in section 2, by recalling the foundations of Chaplygin gas Braneworld inflation and in particular, the modified *Friedmann* equation and various perturbation spectrum parameters are given. In the section 3, we present our results for an exponential potential, and show that for some values of the number of e-folding  $N$ , the inflation parameters are in good agreement with recent WMAP7 data. The conclusions are given in the last section.

## 2 Chaplygin gas Braneworld inflation

### 2.1 Slow-Roll approximation

We start this section by recalling briefly some foundations of *Randall-Sundrum* type 2 Braneworld model. In this scenario, our universe is considered as a 3-brane embedded in *five*-dimensional anti-de Sitter space-time (*AdS5*) where gravitation can propagate through a supplementary dimension. One of the most relevant consequences of this model is the modification of the *Friedmann* equation for energy density of the order of the brane tension or higher. In the case where the matter in the brane is

dominated by the generalized Chaplygin gas (GCG), the gravitational *Einstein* equations lead to the modified *Friedmann* equation on the brane [7, 12]:

$$H^2 = \frac{8\pi}{3m_p^2} \left( A + \rho_\phi^{(\alpha+1)} \right)^{\left( \frac{1}{\alpha+1} \right)} \left( 1 + \frac{(A + \rho_\phi^{(\alpha+1)})^{\frac{1}{\alpha+1}}}{2\lambda} \right), \quad (4)$$

where  $\rho_\phi = \frac{\dot{\phi}^2}{2} + V(\phi)$  and  $V(\phi)$  is the scalar field potential responsible of inflation,  $\lambda$  is the brane tension and  $m_p$  is the *Planck* mass.

One has also a second inflationary relation given by the *Klein-Gordon* Eq. governing the dynamic of the scalar field  $\phi$ :

$$\ddot{\phi} + 3H\dot{\phi} + V' = 0, \quad (5)$$

where  $\dot{\phi} = \frac{\partial\phi}{\partial t}$ ,  $\ddot{\phi} = \frac{\partial^2\phi}{\partial t^2}$  and  $V' = \frac{\partial V}{\partial\phi}$ .

In the following, we will consider the case where  $\alpha = 1$ . During inflation, the energy density associated to the scalar field is comparable to the scalar potential, i.e.  $\rho_\phi \simeq V(\phi)$ . Here we shall introduce the well known slow-roll conditions, i.e.  $\dot{\phi}^2 \ll V(\phi)$  and  $\ddot{\phi} \ll H\dot{\phi}$ . Note that, perturbations on the brane are, generally, coupled to the bulk metric perturbations. This make the situation more complicated. However, on large scales on the brane, the density perturbations decouple from the bulk metric perturbations and the extreme slow-roll limit (de Sitter) become applicable for RS-type II model[15]. In this case, the *Friedmann* equation reduces to

$$H^2 = \frac{8\pi}{3m_p^2} \sqrt{A + V^2} \left( 1 + \frac{(A + V^2)^{\frac{1}{2}}}{2\lambda} \right), \quad (6)$$

In the some context, one can also consider the slow-roll parameters to study the spectrum of the perturbation[15]. The two first parameters are given for GCG model by

$$\varepsilon = \frac{m_p^2}{16\pi} \frac{V V'^2}{(A + V^2)^{\frac{3}{2}}} \frac{\left( 1 + \frac{(A + V^2)^{\frac{1}{2}}}{\lambda} \right)}{\left( 1 + \frac{(A + V^2)^{\frac{1}{2}}}{2\lambda} \right)^2}, \quad (7)$$

$$\eta = \frac{m_p^2}{8\pi} \frac{V''}{(A + V^2)^{\frac{1}{2}}} \frac{1}{\left( 1 + \frac{(A + V^2)^{\frac{1}{2}}}{2\lambda} \right)}, \quad (8)$$

where  $V' = \frac{\partial V}{\partial\phi}$  and  $V'' = \frac{\partial^2 V}{\partial\phi^2}$ . Note that during inflation, the conditions  $\varepsilon \ll 1$  and  $|\eta| \ll 1$  are satisfied.

On the other hand, one can derive the number of e-folding as

$$N = -\frac{8\pi}{m_p^2} \int_{V_*}^{V_{end}} \frac{\sqrt{A + V^2}}{V'^2} \left( 1 + \frac{(A + V^2)^{\frac{1}{2}}}{2\lambda} \right) dV, \quad (9)$$

where  $V_*$  and  $V_{end}$  are the values of the potentials at the horizon exit and the end of inflation, respectively.

## 2.2 Chaplygin gas perturbation spectrum

The spectrum of the inflationary perturbations is produced by quantum fluctuations of the scalar field around its homogeneous background values. Thus, the power spectrum of the curvature perturbations  $P_R(k)$  is given in the slow-roll approximation by the following expression[15]

$$P_R(k) = \left( \frac{H^2}{2\pi\dot{\phi}} \right)^2 \quad (10)$$

$$= \frac{128\pi}{3m_p^6} \frac{(A+V^2)^{\frac{3}{2}}}{V'^2} \left( 1 + \frac{(A+V^2)^{\frac{1}{2}}}{2\lambda} \right)^3, \quad (11)$$

Note that, the power spectrum of the curvature perturbations  $P_R(k)$  allows us to define the scalar spectral index  $n_s$  as[16]

$$n_s - 1 = \frac{d \ln P_R(k)}{d \ln k} \quad (12)$$

$$= \frac{m_p^2}{8\pi(A+V^2)^{\frac{1}{2}} \left( 1 + \frac{(A+V^2)^{\frac{1}{2}}}{2\lambda} \right)} \times \left( -3 \frac{VV'^2}{(A+V^2)} \frac{\left( 1 + \frac{(A+V^2)^{\frac{1}{2}}}{\lambda} \right)}{\left( 1 + \frac{(A+V^2)^{\frac{1}{2}}}{2\lambda} \right)} + 2V'' \right). \quad (13)$$

Another important inflationary spectrum parameter is the amplitude of the tensorial perturbations  $P_g(k)$ , describing the primordial gravitational wave perturbations produced by a period of extreme slow-roll inflation, which is defined by[17]

$$P_g(k) = \frac{64\pi}{m_p^2} \left( \frac{H}{2\pi} \right)^2 F^2(x) \quad (14)$$

$$= \frac{128}{3m_p^4} \sqrt{A+V^2} \left( 1 + \frac{(A+V^2)^{\frac{1}{2}}}{2\lambda} \right) F^2(x) \quad (15)$$

where  $x = Hm_p \sqrt{\frac{3}{4\pi\lambda}}$  and  $F^2(x) = (\sqrt{1+x^2} - x^2 \sinh^{-1}(\frac{1}{x}))^{-1}$ . Note that in the high-energy limit ( $\sqrt{A+V^2} \gg \lambda$ ), we have  $F^2(x) \approx \frac{3}{2}x = \frac{3}{2} \frac{\sqrt{A+V^2}}{\lambda}$  and in the low-energy limit ( $\sqrt{A+V^2} \ll \lambda$ ),  $F^2(x) \approx 1$ . Therefore, we recover the standard 4D results at the low-energy limit as expected.

The ratio of tensor to scalar perturbations and the running of the scalar index are presented respectively by

$$r(k) = \left( \frac{P_g(k)}{P_R(k)} \right)_{|k=k_*} \quad (16)$$

$$= \left( \frac{\frac{m_p^2}{\pi} \frac{V'^2 F^2(x)}{(A+V^2) \left( 1 + \frac{(A+V^2)^{\frac{1}{2}}}{2\lambda} \right)^2}} \right)_{|k=k_*} \quad (17)$$

Here,  $k_*$  correspond to the case  $k = Ha$ ; the value when the universe scale crosses the *Hubble* horizon during inflation

$$\frac{dn_s}{d \ln k} = \frac{m_p^2}{4\pi} \frac{V'}{\sqrt{A+V^2}} \frac{1}{\left( 1 + \frac{(A+V^2)^{\frac{1}{2}}}{2\lambda} \right)} \left( 3 \frac{\partial \varepsilon}{\partial \phi} - \frac{\partial \eta}{\partial \phi} \right) \quad (18)$$

In the next section, we consider an exponential potential to evaluate all the previous spectrum parameters describing Braneworld inflation.

### 3 Exponential potential in high energy limit

The exponential potential was studied in various occasions, for example the authors of ref.[18] have shown that inflation becomes possible in Braneworld model for a class of potentials ordinarily too steep to support inflation. This type of potential was also used for tachyonic inflation on the brane[19] and recently for tachyonic Chaplygin gas inflation on the brane[8]. Here we consider an exponential potential of the form

$$V(\phi) = V_0 \exp\left(-\frac{\beta}{m_p}\phi\right) \quad (19)$$

where  $\beta$  and  $V_0$  are constant.

In the present case, we apply this potential to derive and study the behaviour of various spectrum parameters in the Braneworld Chaplygin gas scenario. In the high-energy limit; i.e.  $\sqrt{A+V^2} \gg \lambda$ , all the inflationary parameters will be simplified. In this case, the slow-roll parameters Eqs.(7,8) become

$$\varepsilon = \frac{\beta^2}{4\pi} \frac{\lambda V^3}{(A+V^2)^2}, \quad (20)$$

$$\eta = \frac{\beta^2}{4\pi} \frac{\lambda V}{(A+V^2)}. \quad (21)$$

This give new expressions for the scalar spectral index

$$n_s - 1 = \frac{\beta^2}{4\pi} \frac{\lambda V}{(A+V^2)} \left(-6 \frac{V^2}{(A+V^2)} + 2\right), \quad (22)$$

and for the ratio of tensor to scalar perturbations

$$r = \frac{6\beta^2}{\pi} \frac{\lambda V^2}{(A+V^2)^{\frac{3}{2}}}. \quad (23)$$

On the other hand, the running of the scalar index for Chaplygin gas model becomes

$$\frac{dn_s}{d \ln k} = \frac{\beta^4}{8\pi^2} \frac{\lambda^2 V^2}{(A+V^2)^4} (-A^2 + 9AV^2 - 2V^4). \quad (24)$$

Finally, from Eq.(9), one can deduce the following expression for the e-folding number in the high energy limit

$$N = -\frac{4\pi}{\lambda\beta^2} \left( V_{end} - V_* + A \left( \frac{1}{V_*} - \frac{1}{V_{end}} \right) \right) \quad (25)$$

By combining Eqs.(20) and (21), we obtain

$$\eta = \frac{(A+V^2)}{V^2} \varepsilon. \quad (26)$$

The last equation shows that  $\eta \succ \varepsilon$ , then inflation can end at  $\eta = 1$ . Thus, the Eq.(26) implies that

$$A = V_{end}^2 \left( \frac{1 - \varepsilon_{end}}{\varepsilon_{end}} \right) \quad (27)$$

We see then that, the parameter  $A$  can be expressed in terms of  $V_{end}$  and  $\varepsilon_{end}$ . Notice as well, that since  $A \succ 0$  then  $\varepsilon_{end} \prec 1$ .

Inserting the expression of  $A$  (Eq.(27)) in Eq.(21), we obtain

$$V_{end} = \frac{\lambda\beta^2}{4\pi} \varepsilon_{end}. \quad (28)$$

According to Eq.(25), we find

$$V_* = \mu V_{end}, \quad (29)$$

where

$$\mu = \frac{\frac{N}{\varepsilon_{end}} + \varepsilon_{end} + \sqrt{\left(\frac{N}{\varepsilon_{end}} + \varepsilon_{end}\right)^2 + 4\left(\frac{1-\varepsilon_{end}}{\varepsilon_{end}}\right)}}{2}. \quad (30)$$

We signal that, in the limit where  $A \rightarrow 0$ , we obtain

$$\varepsilon_{end} = 1 \quad (31)$$

$$\mu = N + 1 \quad (32)$$

Consequently, we can find all the inflationary parameters in terms of  $V_*$  using Eqs.(28,29)

$$n_s - 1 = \frac{\mu}{\varepsilon_{end} \left(\mu^2 + \frac{1-\varepsilon_{end}}{\varepsilon_{end}}\right)} \left( \frac{-6\mu^2}{\left(\mu^2 + \frac{1-\varepsilon_{end}}{\varepsilon_{end}}\right)} + 2 \right). \quad (33)$$

The equation (33) shows that the scalar spectral index  $n_s$  depends only on  $N$  and  $\varepsilon_{end}$ . In the limit where  $A \rightarrow 0$ , we get

$$n_s - 1 = -\frac{4}{N + 1}. \quad (34)$$

This result was first pointed out in ref.[18].

Note that for  $N = 50$ , we have  $n_s \simeq 0.9215$  whereas for  $N = 55$ ,  $n_s \simeq 0.9285$  and for  $N = 60$ ,  $n_s \simeq 0.9344$ . These values are outside the range given by WMAP7 observation[20], since

$$0.963 \preceq n_s \preceq 1.002 \quad (95\%CL) \quad (35)$$

Thus, generally the e-folding number must be very large.

On the other hand, in the Chaplygin gas model, we have also derived the ratio of tensor to scalar perturbations, which is given by

$$r = \frac{24\mu^2}{\varepsilon_{end} \left(\mu^2 + \frac{1-\varepsilon_{end}}{\varepsilon_{end}}\right)^{\frac{3}{2}}}. \quad (36)$$

Similarly, in the limit where  $A \rightarrow 0$ ,  $r$  becomes

$$r = \frac{24}{N + 1}. \quad (37)$$

As for  $n_s$ , we recover here the results of ref.[18]. The Eq.(37) denotes that the number of e-folding  $N$  must satisfy the inequality  $N \succ 65$  in order for  $r$  to be consistent with WMAP7 since[20]

$$r < 0.36 \quad (95\%CL) \quad (38)$$

In the original Chaplygin exponential Braneworld inflation model ( $\alpha = 1$ ), the running of the scalar index is

$$\frac{dn_s}{d \ln k} = \frac{2\mu^2}{\varepsilon_{end}^2 \left(\mu^2 + \frac{1-\varepsilon_{end}}{\varepsilon_{end}}\right)^4} \left( -2\mu^4 + 9\mu^2 \left(\frac{1-\varepsilon_{end}}{\varepsilon_{end}}\right) - \left(\frac{1-\varepsilon_{end}}{\varepsilon_{end}}\right)^2 \right). \quad (39)$$

Similarly to the previous case, in the limit where  $A \rightarrow 0$ , we get

$$\frac{dn_s}{d \ln k} = -\frac{4}{(N + 1)^2}. \quad (40)$$

This equation shows that the running of the scalar index is small and negative for any value of e-folding number  $N$ , which correspond to sufficient inflation.

To confront simutaniously the observables  $n_s$ ,  $r$  and  $\frac{dn_s}{d \ln k}$  with observations, we study the relative variation of these parameters. Fig.1, shows that the paramater  $r$  behaves as a decreasing function with

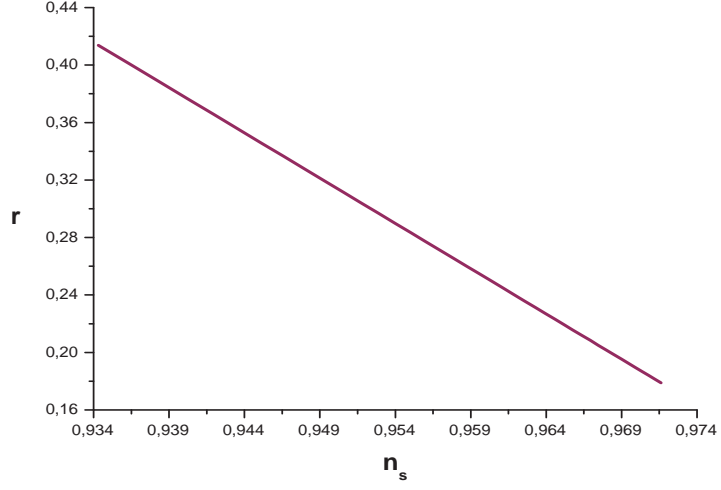


Figure 1:  $r$  vs  $n_s$  for exponential potential in Chaplygin gas Braneworld inflation

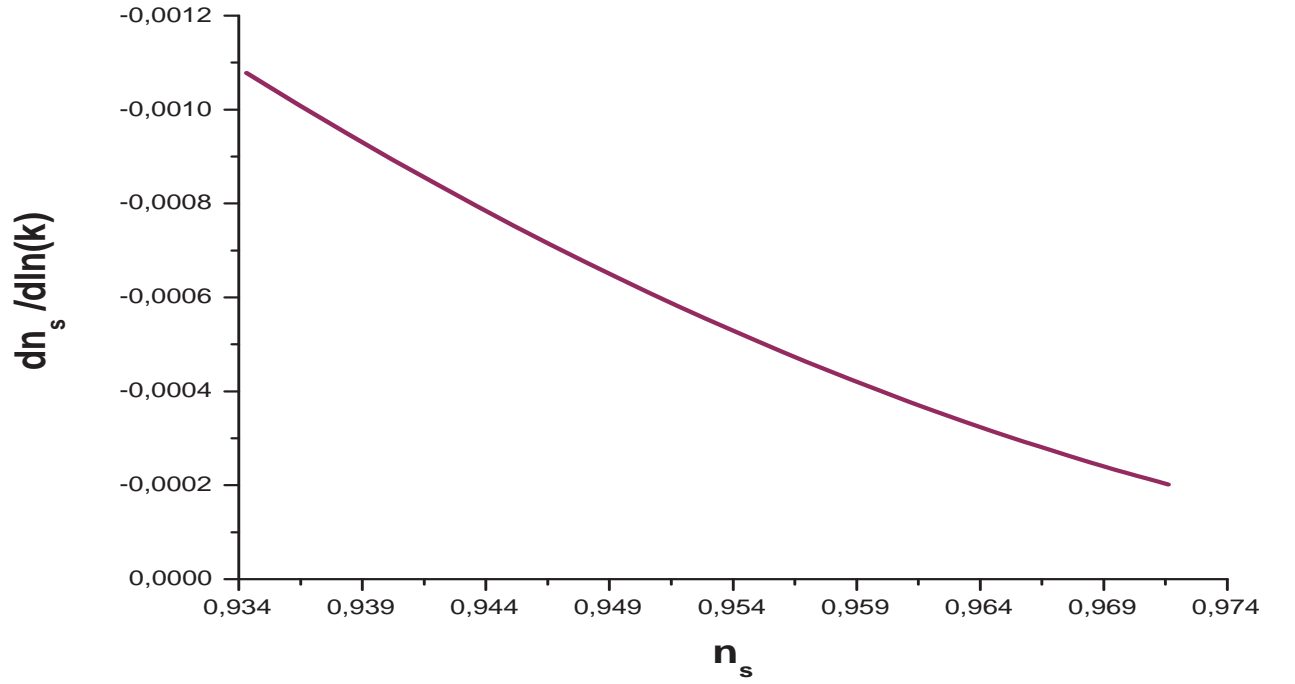


Figure 2:  $\frac{dn_s}{d\ln(k)}$  vs  $n_s$  for exponential potential in Chaplygin gas Braneworld inflation

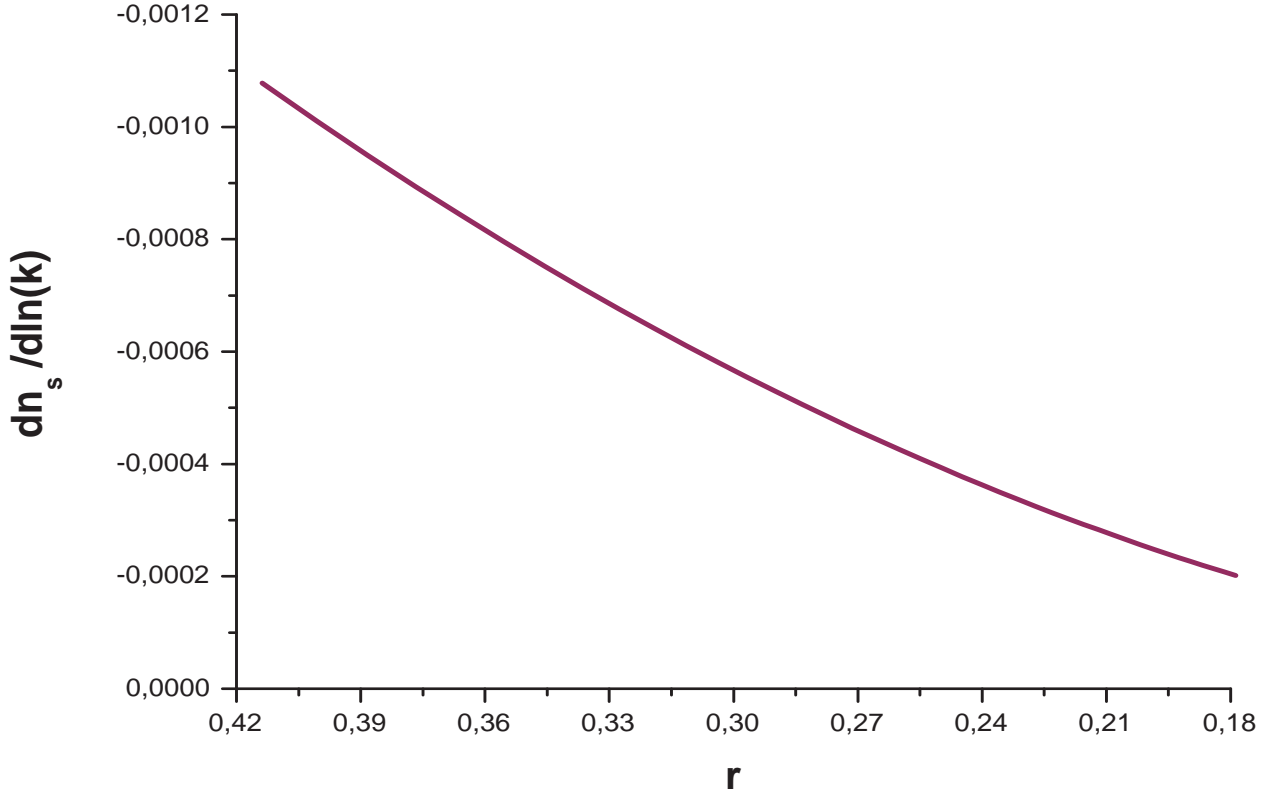


Figure 3:  $\frac{dn_s}{d\ln k}$  vs  $r$  for exponential potential in Chaplygin gas Braneworld inflation

respect to  $n_s$ . We observe that a large domain of variations of  $r$  is consistent with WMAP7 data. We note also that, the scalar spectral index values are in good agreement with the observations for large  $N$ . In Fig.2, we plot the running of the scalar index  $\frac{dn_s}{d\ln k}$  versus the scalar spectral index  $n_s$ . We show that the range, in which the scalar spectral index  $n_s$  is consistent with the recent data, corresponding to negligible running. In the last figure, we show that  $\frac{dn_s}{d\ln k}$  is a decreasing function according to  $r$ . We observe in this case that, for any values of  $r$  allowed by the observations (Eq.38), the running of the scalar index  $\frac{dn_s}{d\ln k}$  is small and negative. This result allows us to consider a negligible running which justifies our choice of observable values for the inflationary parameters.

## 4 Conclusion

In this paper, we have studied a Chaplygin gas model in the framework of Braneworld inflation using an exponential potential. We have adopted here the slow roll approximation in the high-energy limit to derive all inflationary spectrum perturbation parameters in the particular case where  $\alpha = 1$ . See Eq.(1). In this way, we have shown that the inflationary parameters depend only on the e-folding number  $N$  and the final value of the slow roll parameter  $\varepsilon_{end}$ . We have also shown that, for small and negligible running of the scalar spectral index  $\frac{dn_s}{d\ln k}$ , the inflationary parameters are in good agreement with WMAP7 data. On the other hand, we have analyzed the limit  $A \rightarrow 0$ , where the parameters of inflation depend only



on  $N$  and the compatibility with the observations remains conditioned by the values of the number of e-folding  $N$ .

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